NAG Toolbox for MATLAB

d03ne

1 Purpose

d03ne computes average values of a continuous function of time over the remaining life of an option. It is used together with d03nd to value options with time-dependent parameters.

2 Syntax

```
[phiav, ifail] = d03ne(t0, tmat, td, phid, 'ntd', ntd)
```

3 Description

d03ne computes the quantities

$$\phi(t_0), \qquad \hat{\phi} = \frac{1}{T-t_0}\int_{t_0}^{T} \phi(\zeta)\, d\zeta, \qquad \bar{\phi} = \left(\frac{1}{T-t_0}\int_{t_0}^{T} \phi^2(\zeta)\, d\zeta\right)^{1/2}$$

from a given set of values **phid** of a continuous time-dependent function $\phi(t)$ at a set of discrete points **td**, where t_0 is the current time and T is the maturity time. Thus $\hat{\phi}$ and $\bar{\phi}$ are first and second order averages of ϕ over the remaining life of an option.

The function may be used in conjunction with d03nd in order to value an option in the case where the risk-free interest rate r, the continuous dividend q, or the stock volatility σ is time-dependent and is described by values at a set of discrete times (see Section 8.2). This is illustrated in Section 9.

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **t0 – double scalar**

The current time t_0.

Constraint: $\mathbf{td}(1) \leq \mathbf{t0} \leq \mathbf{td}(\mathbf{ntd})$.

2: **tmat – double scalar**

The maturity time T.

Constraint: $\mathbf{td}(1) \leq \mathbf{tmat} \leq \mathbf{td}(\mathbf{ntd})$.

3: **td(ntd) – double array**

The discrete times at which ϕ is specified.

Constraint: $\mathbf{td}(1) < \mathbf{td}(2) < \cdots < \mathbf{td}(\mathbf{ntd})$.

4: **phid(ntd) – double array**

$\mathbf{phid}(i)$ must contain the value of ϕ at time $\mathbf{td}(i)$, for $i = 1, 2, \ldots, \mathbf{ntd}$.

5.2 Optional Input Parameters

1: **ntd – int32 scalar**

Default: The dimension of the arrays **td**, **phid**. (An error is raised if these dimensions are not equal.)

the number of discrete times at which ϕ is given.

Constraint: **ntd** ≥ 2.

5.3 Input Parameters Omitted from the MATLAB Interface

work, lwork

5.4 Output Parameters

1: **phiav**(3) **– double array**

phiav(1) contains the value of ϕ interpolated to t_0, **phiav**(2) contains the first-order average $\hat{\phi}$ and **phiav**(3) contains the second-order average $\bar{\phi}$, where:

$$\hat{\phi} = \frac{1}{T-t_0}\int_{t_0}^{T} \phi(\zeta) d\zeta, \qquad \bar{\phi} = \left(\frac{1}{T-t_0}\int_{t_0}^{T} \phi^2(\zeta)\, d\zeta\right)^{1/2}.$$

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **t0** lies outside the range [**td**(1), **td**(**ntd**)],
or **tmat** lies outside the range [**td**(1), **td**(**ntd**)],
or **ntd** < 2,
or **td** badly ordered,
or **lwork** $< 9 \times$ **ntd** $+ 24$.

ifail $= 2$

Unexpected failure in internal call to e01ba or e02bb.

7 Accuracy

If $\phi \in C^4[t_0, T]$ then the error in the approximation of $\phi(t_0)$ and $\hat{\phi}$ is $O(H^4)$, where $H = \max_i (T(i+1) - T(i))$, for $i = 1, 2, \ldots,$ **ntd** $- 1$. The approximation is exact for polynomials of degree up to 3.

The third quantity $\bar{\phi}$ is $O(H^2)$, and exact for linear functions.

8 Further Comments

8.1 Timing

The time taken is proportional to **ntd**.

8.2 Use with d03nd

Suppose you wish to evaluate the analytic solution of the Black–Scholes equation in the case when the risk-free interest rate r is a known function of time, and is represented as a set of values at discrete times. A call to d03ne providing these values in **phid** produces an output array **phiav** suitable for use as the argument **r** in a subsequent call to d03nd.

Time-dependent values of the continuous dividend Q and the volatility σ may be handled in the same way.

8.3 Algorithmic Details

The **ntd** data points are fitted with a cubic B-spline using the function e01ba. Evaluation is then performed using e02bb, and the definite integrals are computed using direct integration of the cubic splines in each interval. The special case of $T = t_o$ is handled by interpolating ϕ at that point.

9 Example

```
t0 = 0;
tmat = 0.4166667;
td = [0;
     0.1;
     0.2;
     0.3;
     0.4;
     0.5];
phid = [0.1;
     0.11;
     0.12;
     0.13;
     0.14;
     0.15];
[phiav, ifail] = d03ne(t0, tmat, td, phid)
```

```
phiav =
    0.1000
    0.1208
    0.1214
ifail =
           0
```